Ionospheric electrodynamics and its coupling to the magnetosphere

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Outline

(1) Fluid description \((B,V)\) for partially ionized system

  - Plasma Equation of motions
  - Generalized Ohm’s law

(2) M-I coupling via shear Alfven wave

  - magnetospheric dynamo
  - wave generation and propagation and their relation to velocity shear
  - atmospheric dynamo
  - 3D current closure inside ionosphere
Fundamental evolution equations of physical quantities $Q_k$

$$\frac{\partial}{\partial t} Q_k = F_k (Q_1, Q_2, Q_3, \ldots)$$

$$Q_k = B, E, J, V, V_n$$
What’s drive What in the weakly ionized system?

evolution of B-field
\[ \frac{\partial}{\partial t} \mathbf{B} = - (\nabla \times \mathbf{E}) \]

evolution of E-field
\[ \frac{\partial}{\partial t} \mathbf{E} = \frac{(\nabla \times \mathbf{B}) - \mu_0 \mathbf{j}}{\varepsilon_0 \mu_0} \]

evolution of current density
\[ \frac{\partial}{\partial t} \mathbf{j} = ? \]

evolution of plasma velocity
\[ \frac{\partial}{\partial t} \mathbf{V} = ? \]

evolution of neutral velocity
\[ \frac{\partial}{\partial t} \mathbf{V}_n = ? \]
momentum equations in the collisional system

for ion fluid

\[ m_i n_i \frac{\partial}{\partial t} v_i = e n_i (E + v_i \times B) - \nabla \cdot p_i - m_i v_{in} n_i (v_i - v_n) - m_e v_{ei} n_i (v_i - v_e) \]  

(1)

for electron fluid

\[ m_e n_e \frac{\partial}{\partial t} v_e = -e n_e (E + v_e \times B) - \nabla \cdot p_e - m_e v_{en} n_e (v_e - v_n) + m_e v_{ei} n_i (v_i - v_e) \]  

(2)

for neutral fluid

\[ \rho_n \left[ \frac{\partial v_n}{\partial t} + (v_n \cdot \nabla) v_n \right] = -\nabla p_n - \rho_n v_{ni} (v_n - V_i) - \rho_n v_{ne} (v_n - V_e) + F_n \]  

(3)
One fluid–description for global phenomena
(Hall MHD approximation)

\[ j = en \left( v_i - v_e \right) \]

\[ \mathbf{v} = \frac{m_i v_i + m_e v_e}{m_i + m_e} \]

\[ \rho = \left( m_i + m_e \right) n \]

\[ m_i \gg m_e \]

\[ v_i = \mathbf{V} + \left( \frac{m_e}{\rho e} \right) j \]

\[ v_e = \mathbf{V} - \left( \frac{m_i}{\rho e} \right) j \]
One-fluid description of ion and electron (motion and current)

Equation of motion (mass weighted form of velocity evolution)

For ion:
\[
m_i \frac{d}{dt} \left[ V + \left( \frac{m_e}{\rho e} \right) j \right] = e_n (E + V \times B) + \left( \frac{m_e}{m_i + m_e} \right) j \times B - m_i n v_{in} \left[ V - v_n - \left( \frac{m_e}{\rho e} \right) j \right] - \left( \frac{m_i n V_{ic}}{e} \right) j - \nabla p_i
\]

For electron:
\[
m_e \frac{d}{dt} \left[ V - \left( \frac{m_i}{\rho e} \right) j \right] = -e_n (E + V \times B) + \left( \frac{m_i}{m_i + m_e} \right) j \times B - m_e n v_{en} \left[ V - v_n + \left( \frac{m_i}{\rho e} \right) j \right] - \left( \frac{m_e n V_{ie}}{e} \right) j - \nabla p_e
\]

Current density (charge weighted form of velocity evolution)

Ionic current:
\[
e_n \frac{d}{dt} \left[ V + \left( \frac{m_e}{\rho e} \right) j \right] = \left( \frac{e^2 n}{m_i} \right) (E + V \times B) + e \left( \frac{m_e}{m_i} \right) j \times B - e n v_{in} \left[ V - v_n - \left( \frac{m_e}{\rho e} \right) j \right] - \nabla p_i
\]

Electronic current:
\[
-e_n \frac{d}{dt} \left[ V - \left( \frac{m_i}{\rho e} \right) j \right] = \left( \frac{e^2 n}{m_e} \right) (E + V \times B) - e \left( \frac{m_i}{m_i + m_e} \right) j \times B + e n v_{en} \left[ V - v_n + \left( \frac{m_i}{\rho e} \right) j \right] + \nabla p_e
\]

Ion: main portion of plasma dynamics  Electron: main portion of current carrier
Generalized Equation of motion for one-fluid plasma

\[ \rho \frac{\partial}{\partial t} \mathbf{V} = \mathbf{j} \times \mathbf{B} - \nabla P - n \left( m_i \mathbf{v}_{in} + m_e \mathbf{v}_{en} \right) \left( \mathbf{V} - \mathbf{V}_n \right) - m_e \left( \mathbf{v}_{in} - \mathbf{v}_{en} \right) \frac{\mathbf{j}}{e} \]

- inertial force
- pressure gradient
- friction force along \( j \)
- Ampere force
- momentum exchange between neutral and charged particles

Generalized Ohm’s law

\[ \frac{\partial}{\partial t} \mathbf{j} = \varepsilon_0 \omega_{pe}^2 \left( \mathbf{E} + \text{emf} - \mathbf{\widetilde{R}} \cdot \mathbf{j} \right) \]

- electromotive e-field
- reactive-field
- convection electric field
- am-bipolar field

\[ \text{emf} = \mathbf{V} \times \mathbf{B} + \frac{B \left( \mathbf{v}_{in} - \mathbf{v}_{en} \right)}{\Omega_e} \left( \mathbf{V} - \mathbf{V}_n \right) - \frac{\nabla P_e}{ne} \]

- electric field induced by momentum exchange effect

\[ \mathbf{\widetilde{R}} \cdot \mathbf{j} = \left( \frac{B}{ne} \right) \left[ \frac{\mathbf{v}_{en} + \mathbf{v}_{ei}}{\Omega_e} \right] \mathbf{j} + \left( \mathbf{j} \times \mathbf{\hat{b}} \right) \]

- resistive e-field
- Hall e-field
Distribution of collision frequency

controls the momentum exchange between neutral and charged particles

From Song, et al., [2001]
Equation for motion of plasma (dominant)

\[ \frac{\partial \mathbf{V}}{\partial t} = -\mathbf{v}_{in} (\mathbf{V} - \mathbf{V}_n) + \frac{\mathbf{j} \times \mathbf{B}}{\rho} + \frac{B}{\rho} \left( \frac{\mathbf{v}_{en}}{\Omega_e} \right) \mathbf{j} - \frac{\nabla P}{\rho} \]

- Inertial force
- Momentum exchange between ion and neutrals
- Ampere force
- Friction force along \( \mathbf{j} \)
- Pressure gradient force

- Important \( \omega > \sim \nu_{in} \)
- Dominant in the ionosphere
- Everywhere important
- Important below 80 km

Important near plasma sheet
Force balance Equation below

**E-layer**

\[ m_i v_{in} \gg m_e v_{en} \quad v_{in} \ll v_{en} \]

\( \omega \ll v_{in} \)

With cold plasma plasma approximation

\[ \mathbf{0} \approx -v_{in}(\mathbf{V} - \mathbf{V}_n) + \frac{j \times \mathbf{B}}{\rho} + \frac{B}{\rho} \left( \frac{v_{en}}{\Omega_e} \right) j \]

- inertial momentum exchange between ion and neutrals
- Ampere force
- friction force along \( j \)

**Figure 1.** Collision frequencies and gyrofrequencies as functions of altitude at local noon at 75° latitude. They are determined based on the observations/laboratory experiments and the formula of Kelley [1989]. The anisotropy in the collision frequencies becomes unimportant above 80 km. More detailed discussion on the relationships among these quantities and anisotropies is given by Richmond [1995].

- dominant in the ionosphere
- everywhere important
- important below 80km
Ionospheric current is a force balance current

\[ \mathbf{j} \times \mathbf{B} = \frac{en}{k_{en}} \left\{ k_{in}^{-1} \left[ \frac{k_{en}^2}{1 + k_{en}^2} \left( \mathbf{V} - \mathbf{V}_n \right) \right] + \frac{k_{en}^2}{1 + k_{en}^2} \left[ k_{in}^{-1} \left( \mathbf{V} - \mathbf{V}_n \right) \times \mathbf{B} \right] \right\} \]

For 90km~140km

\[ \mathbf{j} \times \mathbf{B} \approx k_{in}^{-1} en \left( \mathbf{V} - \mathbf{V}_n \right) \]

Important only for 80km

Atmospheric dynamo through collision via electron
(dynamo region-> 70~90km)

Atmospheric dynamo through collision via ion
(dynamo region-> 90~140km)
plasma – neutral interaction

\[ \mathbf{j} \times \mathbf{B} \approx \rho \mathbf{v}_{in} (\mathbf{V} - \mathbf{u}_n) \]

**Equivalence of electromagnetic load and mechanical load**

In the neutral frame

\[ (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{V}^* \approx \rho \mathbf{v}_m |\mathbf{V}^*|^2 \quad \mathbf{V}' \equiv \mathbf{V} - \mathbf{u}_n \]

Work done by Ampere force = heating by plasma

In the rest (plasma) frame

Work done by Ampere force = work done by momentum exchange by collision

\[ (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} \equiv \rho \mathbf{v}_{in} (\mathbf{V} - \mathbf{u}_n) \cdot \mathbf{v} = \mathbf{j} \cdot \mathbf{E} \]

\[ \mathbf{j} \cdot \mathbf{E} > 0 \quad \text{for} \quad \mathbf{v} > \mathbf{u}_n \]

Atmospheric load

\[ \mathbf{j} \cdot \mathbf{E} < 0 \quad \text{for} \quad \mathbf{v} < \mathbf{u}_n \]

Atmospheric dynamo

Electromagnetic energy is converted into mechanical energy

Electromagnetic energy is generated from mechanical energy
Atmospheric dynamo

- **Neutral wind**
- **Plasma flow (within the dynamo region)**
- **Magnetic perturbation, current**
- **Electric field (within the dynamo region)**
- **Plasma flow outside dynamo region**
- **To the magnetosphere**

**Generalized Ohm’s law**

**Momentum exchange by ion-neutral collision**

**Ampere force (magnetic tension acting plasma)**

**Non-zero curl E (different E within and outside dynamo region along B₀)**

**Alfvén wave generation**
Atomospheric dynamo

\[(j \times B) \cdot V \approx k_{in}^{-1}en(V - V_n) \cdot V\]

Work done by Ampere force acting on ion  
Work done by momentum exchange between ion and neutral on ion

Case (A) if

\[(j \times B) \cdot V \approx k_{in}^{-1}en(V - V_n) \cdot V > 0\]

Ampere force can accelerate neutrals through collision (atmospheric load)

Case (B) if

\[(j \times B) \cdot V \approx k_{in}^{-1}en(V - V_n) \cdot V < 0\]

Neutral particle can produces Ampere force through Collision (electromagnetic energy is produced by kinetic energy of Neutral particles) (atmospheric dynamo)
Generalized Ohm’s law (dominant)

Current evolution equation

\[
\frac{\partial}{\partial t} \mathbf{j} = \varepsilon_0 \omega^2_{pe} \left( \mathbf{E} + \mathbf{emf} - \mathbf{R} \cdot \mathbf{j} \right)
\]

Electromotive e.m.f. e-field

\[
\overline{emf} = \mathbf{V} \times \mathbf{B} - B \left( \frac{V_{en}}{\Omega_e} \right) (\mathbf{V} - \mathbf{v}_n)
\]

Convection e-field

\[
\mathbf{R} \cdot \mathbf{j} = \left( \frac{B}{ne} \right) \left[ \left( \frac{V_{en}}{\Omega_e} \right) \mathbf{j}_\perp + \left( \mathbf{j} \times \hat{\mathbf{b}} \right) \right]
\]

Resistive e-field

Hall e-field

Figure 1. Collision frequencies and gyrofrequencies as functions of altitude at local noon at 75° latitude. They are determined based on the observations/laboratory experiments and the formula of Kelley [1989]. The anisotropy in the collision frequencies become unimportant above 80 km. More detailed discussion on the relationships among these quantities and anisotropies is given by Richmond [1995].
Electromagnetic wave Radiation

**evolution of B-field**

\[
\frac{\partial}{\partial t} \mathbf{B} = - (\nabla \times \mathbf{E})
\]

**evolution of E-field**

\[
\frac{\partial}{\partial t} \mathbf{E} = \frac{(\nabla \times \mathbf{B}) - \mu_0 \mathbf{j}}{\varepsilon_0 \mu_0}
\]

\[
\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} + \nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial}{\partial t} \mathbf{j}
\]

**evolution eq of \( \mathbf{J} \)?**

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**no!! electromagnetic field radiations eq by changing of current**

(like a dipole antenna)

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**we need to know how \( \mathbf{J} \) is produced by electro-dynamics (not -magnetics)**

\[
\frac{\partial}{\partial t} \mathbf{j} = \varepsilon_0 \omega_{pe}^2 \left( \mathbf{E} - \text{emf} - \tilde{\mathbf{R}} \mathbf{j} \right)
\]

\[
\tilde{\mathbf{R}} \mathbf{j} = \left[ \frac{B}{ne} \right] \left[ \left( \frac{\nu_{en} + \nu_{ei}}{\Omega_e} \right) \mathbf{j} + \mathbf{j} \times \hat{\mathbf{b}} \right]
\]
After radiation of plasma waves

\[
\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} + \nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial}{\partial t} \mathbf{j}
\]

\[
\frac{\partial}{\partial t} \mathbf{j} = \varepsilon_0 \omega_p \left( \mathbf{E} + \mathbf{emf} - \left[ \frac{\mathbf{R}}{\mu_0} \right] \left( \frac{\nu_e + \nu_i}{\Omega_e} \right) \mathbf{j} \times \mathbf{e}_B \right)
\]

\[
\mathbf{j} = \frac{(\nabla \times \mathbf{B})}{\mu_0} - \varepsilon_0 \frac{\partial}{\partial t} \mathbf{E}
\]

\[
-\lambda_e^2 \nabla \times (\nabla \times \mathbf{E}) - \omega_{pe}^{-2} \frac{\partial^2}{\partial t^2} \mathbf{E} + \left( \frac{\Omega_e}{\omega_{pe}^2} \right) \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{e}_B) + \left( \frac{\nu_e - \nu_i}{\omega_{pe}^2} \right) \frac{\partial}{\partial t} \mathbf{E} = \mathbf{E} - \mathbf{emf} - \mu_0^{-1} \mathbf{R} \cdot (\nabla \times \mathbf{B})
\]

\[
L \gg \lambda_e
\]

much larger than electron inertial length

\[
T \gg \omega_{pe}^{-1}
\]

much slower than plasma oscillation

\[
\mathbf{E} = \mathbf{emf} + \mu_0^{-1} \mathbf{R} \cdot (\nabla \times \mathbf{B})
\]

after electromagnetic wave radiation

balance equation among \(\mathbf{E}\), \(\mathbf{emf}\) and reactive fields: (Steady Ohm’s law) in \((\mathbf{B}, \mathbf{V})\) scheme is established
Ohm’s law in ion-collisional system

Eq of motion (force balance)
\[ \mathbf{j} \times \mathbf{B} \simeq k_{in}^{-1}en(\mathbf{V} - \mathbf{V}_n) \]

Generalized Ohm’s law
\[ \mathbf{E} = \mathbf{emf} + \mathbf{\bar{R}} \cdot \mathbf{j} \]

- Convection e-field
  \[ \mathbf{emf} = -\mathbf{V} \times \mathbf{B} \]
- Hall e-field
  \[ \mathbf{\bar{R}} \cdot \mathbf{j} \simeq \left( \frac{\mathbf{B}}{ne} \right)(\mathbf{j} \times \hat{\mathbf{b}}) \]
- Ion-collisional resistive e-field
  \[ = k_{in}^{-1}B(v_i - v_n) \]

Ohm’s law in the rest frame
\[ \mathbf{E} = \mathbf{emf} + \mathbf{\bar{R}} \cdot \mathbf{j} \]
\[ \mathbf{E} = -\mathbf{V} \times \mathbf{B} + k_{in}^{-1}B(\mathbf{V} - \mathbf{v}_n) \]

In the ionosphere electric field is induced not only in -VxB (convection electric field) direction but also in the direction of plasma flow !!(very different from magnetosphere)
Magnetospheric Generator

“Generator” region can be defined as a region enable to production of velocity shear (vorticity) both parallel and perpendicular direction to $B_0$ by the mechanical balance.

Parallel shear induces $(\text{div } E_\perp)$

$$\left(\nabla \times \mathbf{v}\right)_\parallel \propto \left(\nabla \cdot E_\perp\right)\hat{e}_B$$

$$\mathbf{v}_\perp = \frac{\mathbf{E} \times \hat{e}_B}{B_0}$$

Perpendicular shear produces $(\text{rot } E_\perp)$

$$\frac{\partial}{\partial s} \mathbf{v}_\perp \propto \left(\nabla \times \mathbf{v}\right)_\perp \propto \left(\nabla \times E_\perp\right)_\perp$$

Always inductive!!

Note: $\nabla \cdot (\nabla \times \mathbf{v}) = 0 \quad \rightarrow \quad \nabla \cdot \left(\nabla \times \mathbf{v}\right)_\parallel = -\nabla \cdot \left(\nabla \times \mathbf{v}\right)_\perp$

Vorticity conservation as like a current

$$\nabla \cdot \mathbf{j} = 0 \quad \rightarrow \quad \nabla \cdot \mathbf{j}_\parallel = -\nabla \cdot \mathbf{j}_\perp$$

Shear Alfvén wave

$$\mathbf{j}_\perp = B_0 \Sigma_A \left(\nabla \times \mathbf{v}\right)_\perp$$

$$\nabla \cdot \mathbf{j}_\perp = 0 \quad \leftrightarrow \quad \nabla \cdot (\nabla \times \mathbf{v}) = 0$$

$$\mathbf{j}_\parallel = \Sigma_A \left(\nabla \times \mathbf{v}\right)_\parallel$$
Induction process: generation and propagation shear Alfven wave

(1) Generator: drives enhanced convection velocity by balance of acceleration by pressure gradient force and deceleration by the magnetic tension force

Alfven Induction loop responsible for A.Y

(2) velocity shear along corresponds to rot(E)
   → development of magnetic field
   → development of tension force
   → acceleration of plasma flow at wave front

\[
\frac{\partial}{\partial s} \mathbf{V}_\perp \propto (\nabla \times \mathbf{E})_\perp \propto \frac{\partial}{\partial t} \mathbf{b}_\perp \rightarrow \mathbf{j} \times \mathbf{B}_0 \rightarrow \rho \frac{\partial}{\partial t} \mathbf{V}_\perp
\]

- role of shear Alfven wave → disappearance of velocity shear along \( B_0 \)
- Generator acts until difference of convection velocity along \( B_0 \) is disappeared

\[
\nabla \cdot \mathbf{S} > 0
\]

\[
(\mathbf{j} \times \mathbf{B}_0) \cdot \mathbf{v}_0 > 0 \quad -\nabla p \cdot \mathbf{v}_0 < 0
\]
Magnetospheric dynamo

Enhanced convection velocity (plasma flow within the dynamo region)

\[ \text{Generalized Ohm's law} \]

Electric field (within the dynamo region)

\[ \text{Non-zero curl } E \text{ (different } E \text{ within and outside dynamo region along } B_0 \text{)} \]

Magnetic perturbation, current

\[ \text{Ampere force (magnetic tension force acting plasma)} \]

Plasma flow outside dynamo region

velocity shear along corresponds to \( \text{rot}(E) \)

\[ \rightarrow \text{development of magnetic field} \]
\[ \rightarrow \text{development of tension force} \]
\[ \rightarrow \text{acceleration of} \]
\[ \text{plasma flow at wave front} \]

\[ \text{To the ionosphere} \]

Alfvén induction loop
Propagation of electromagnetic field inside the ionosphere and production of reflection field

- Ion collision
- Alfven-induction loop

From the magnetosphere

- Velocity shear along $B_0$ corresponds to $\text{rot}(E)_\perp$
- Development of magnetic field
- Development of tension force
- Acceleration of ion flow
- Braking of ion flow by collision
- No braking of electron flow
- Attenuation of $v_i$
- No attenuation of $v_e$

Motion of ion induces two types electric field
- (1) Convection electric field
- (2) Hall electric field

Process (1)

(1) reflection
- Alfven-induction loop

To the magnetosphere

- Plasma flow outside dynamo region
- Amplere force
- Magnetic perturbation, current
- Shear (rot) of e-field along $B$-field
- Equivalent to generation of oppositely directed e-field
- Generalized Ohm's law

Attenuation of $v_i$ through collision
Corresponds to attenuation of type (1) e-field
Propagation of electromagnetic field inside the ionosphere and production of reflection field

Ion collision
Alfven-induction loop

From the magnetosphere

velocity shear along B0 corresponds to \( \text{rot}(E) \perp \)
\( \rightarrow \) development of magnetic field
\( \rightarrow \) development of tension force
\( \rightarrow \) acceleration of ion flow
\( \rightarrow \) braking of ion flow by collision
\( \rightarrow \) no braking of electron flow

attenuation of \( v_i \)
No attenuation of \( v_e \)

Motion of ion induces two types electric field
(1) Convection electric field
(2) Hall electric field

Hall dynamo
Alfven-induction loop

To the magnetosphere

Plasma flow out side dynamo region

Ampere force
Magnetic perturbation, current

This e-filed perpendicular to convection type e-field

Shear of e-field along B0

Generation of e-field in the direction \( v_i \)

Generalized Ohm's law